

**MATH SERIES 2**

**GEOMETRY, Part 3: Volume of Solids**

*25 Minutes*

Distributed by BENCHMARK MEDIA

**FOR USE IN:** Mathematics

**LEVEL:** Grades 7-9

**EDUCATIONAL ADVISOR:** Richard Albero, Math Instructor, Briarcliff Manor High School, MS Educational Psychology, MS Physics

**EDUCATIONAL OBJECTIVES:**

To help students understand:

**the use of standard cubic units to measure volume**

**how to calculate the volume of right prisms: rectangular, circular (cylinder), and triangular**

**how to calculate the volume of a: cone, pyramid, and sphere**

**BACKGROUND INFORMATION:**

**Geometry** is a branch of mathematics that involves studying the shape, size, and position of lines, angles, curves, and figures. The name *geometry* comes from two Greek words meaning *earth* and *to measure*. The earliest uses of geometry included measuring lengths and areas of land. Most scholars believe that the ancient Egyptians and Babylonians were the first people to use geometry extensively.

Geometric shapes fill the world around us. For example, honeybees build their honeycombs in a pattern of hexagons, and earthworms are shaped like cylinders. Most houses and buildings have walls shaped like rectangles, and many bridges have supports shaped like triangles. By knowing something about geometry, we can better understand and appreciate our world.

Geometry has many practical uses. To construct stable and attractive buildings, architects and carpenters, for example, must understand the characteristics of geometric objects. Navigators of airplanes, ships, and spacecraft rely on geometric ideas to chart and follow a set course. Artists, designers, engineers, and photographers also use geometric principles in their work.

## **BEFORE SHOWING THE VIDEO**

In Geometry Part 1, we learned how to calculate *the areas of two dimensional shapes*: parallelograms, trapezoids, and triangles; and the circumference and area of circles.

In Geometry Part 2, we learned how to calculate *the surface areas of solids* (3 dimensional) figures, such as right prisms: rectangular, triangular, and circular (cylinder); and some non-right prisms, such as: four sided pyramids, cones, and spheres. We used the formulas for the area of two dimensional shapes learned in Geometry Part 1 to calculate the two dimensional surface area of various shapes found on the surface area of solids, shapes such as parallelograms, trapezoids, triangles, and circles, and then add up those areas to find the total surface area of the solid object.

In this video, Geometry Part 3, we will be calculating *the volume of solids*, such as the right prisms: rectangular, triangular, and circular (cylinder); and certain non-right prisms, such as: four sided pyramids, cones, and spheres.

## **CONTENT OF THE VIDEO**

### **Standard Units of Measure**

Volume must be measured with a standard size cube. The cube's length, width, & height are all congruent. Each side is a square. The larger the volume of the object to be measured, the larger the standard units of measurement used for the sides of the cube; i.e., centimeters or meters; inches or feet. One standard sized cube, for example, measures 1 inch on all three sides, and is called a cubic inch, and is written as, cu in, or in<sup>3</sup>.

### **Rectangular Right Prism or Box**

The volume of a box is visualized by filling it with standard sized 1 inch cubes. Area of the base is  $A = lw$  (length) (width). The formula  $V = lwh$  ( $h =$  height) or  $V = Ah$  for the volume of the box, is then visually understood. Measurements of an actual box are substituted for the terms in the formulas, and its volume calculated.

### **Triangular Right Prism**

We use a tent as a triangular right prism. To find its volume, we use the same formula as for the volume of a box,  $V = Ah$ . But in this case the  $A$  is a triangular end face, and the height is the length of the tent between triangular end faces. We know the formula for the area of a triangle is  $A = \frac{bh}{2}$

$\frac{bh}{2}$

where  $b =$  the length of the base of the triangle, and  $h$  is the height of the triangle as measured on a line beginning from its apex and ending perpendicular to its base. Measurements are taken and substituted in the formula, resulting in the area of the triangular end face in square feet. That value, multiplied by the height (distance between the triangular end faces of the tent), results in the volume of the tent in cubic feet.

### Circular Right Prism (a Cylinder)

To find the volume of a **circular right prism (a cylinder)**, found in the tent, we measure the diameter of the circular end face, then divide that value by 2 to get the radius. Using the formula for the area of a circle  $A = \pi r^2$ , we substitute the radius measurement, and calculate the area of the circular end face in square inches. Again we use the  $V = Ah$  basic formula for right prisms, substitute  $\pi r^2$  for  $A$ , resulting in  $V = \pi r^2 h$ . Measure the height of the cylinder in inches, substitute in the formula, and we have the volume of the cylinder in cubic inches.

### Non-Right Prism Shapes.

#### Four-sided Pyramid

The formula for the volume of a **four sided pyramid**, is  $V_p = 1/3 Ah$ . Remember that the formula for the volume of a rectangular right prism, a box is  $V_b = Ah$ . So the volume of a pyramid is 1/3 the volume of a rectangular box with the same base area and height, or  $V_p = 1/3 V_b$ . This is demonstrated with a pyramid and box of the same base area and height. The pyramid is filled three times with sand and emptied into the box, exactly filling the box. Measurements of the pyramid are substituted for terms in the formula  $V_p = 1/3 Ah$ .

#### Sphere

The formula for the volume of a **sphere**, is  $V = 4/3 \pi r^3$ . To determine the volume of a scoop (a sphere) of ice cream, the radius is 1.5 inches, that value substituted for  $r$  in the formula and then the volume calculated.

#### Cone

If ice cream were packed into the cone, filling it completely, would the girl on camera enjoy more ice cream – would the cone have more volume – than in a scoop on top of the cone.

The formula for the volume of a cone is  $V_c = 1/3 Ah$ . Remember that the formula for the volume of a circular right prism, a cylinder is  $V_{cy} = Ah$ . So the volume of a cone is 1/3 the volume of a cylinder with the same base area and height, or  $V_c = 1/3 V_{cy}$ . This is demonstrated with a cone and cylinder of the same base area and height. The cone is filled three times with sand and emptied into the cylinder, exactly filling it.. Measurements of the cone are substituted for terms in the formula  $V_c = 1/3 Ah$ ,  $A$ , being the area of the circular end face, equals  $\pi r^2$  and height is the length along the curved side of the cone. The volume of ice cream filling the cone is slightly more than that of the volume of the sphere of ice cream on top of the cone.

### AFTER SHOWING THE VIDEO

The students may be asked to recall and review the following key concepts in the video:

- What is the standard unit of measurement used to calculate volume? (cubes)
- How is the volume of a rectangular right prism, a box, calculated? What is the formula used?

- What was the formula used to find the volume of a triangular right prism, the tent? What measurements were needed?
- What was the formula used to find the volume of circular right prism or cylinder found in the tent? What two measurements were needed to find its volume?
- What is the volume ratio of any size of pyramid and a rectangular right prism, when both have the same base area and height? How would you express that in an equation?
- What is the volume ratio of any size of cone and cylinder, when both have the same base area and height? How would you express that in an equation?
- What single measurement is needed to find the volume of a sphere? What formula is used?

**Math Series 1, consists of 10 videos:**

ALGEBRA: A Piece of Cake Part 1

ALGEBRA: A Piece of Cake Part 2

SLOPES: That's a Bit Steep!

PERCENTAGES: That Make Sense

LINEAR EQUATIONS and Their Graphs: Let's Get It Straight Part 1

LINEAR EQUATIONS and Their Graphs: Let's Get It Straight Part 2

INTEGER OPERATIONS: Into the Negative Zone Part 1 Adding and Subtracting

INTEGER OPERATIONS: Into the Negative Zone Part 2 Multiplying and Dividing

FACTORING IS FANTASTIC Part 1: Common Factors

FACTORING IS FANTASTIC Part 2: Quadratic Trinomials

**Math Series 2, consists of 12 videos:**

PROBABILITY, Parts 1 & 2

RATIOS

TRIGONOMETRY, Parts 1 & 2

STATISTICS Parts 1 & 2

PROBLEM SOLVING Parts 1 & 2

GEOMETRY Parts 1, 2, &3

**BENCHMARK MEDIA** 569 NORTH STATE ROAD, BRIARCLIFF MANOR, NY 10510 TEL:  
914/762-3838, 1/800-438-5564 FAX: 914/762-3895 E-MAIL: benchmedia@aol.com

