TEACHER'S GUIDE

MATH SERIES 2

PROBABILITY, Part 1: Experiment, Theory, & Odds

13 Minutes

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FOR USE IN: Mathematics

LEVEL: Grades 7-9

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EDUCATIONAL OBJECTIVES:
To help the students understand:

- **the concepts of certain and impossible events, and the probability of an event happening which is neither certain nor impossible**
- **small and large number experimental trials with replacement to approximate probability**
- **defining and calculating theoretical probability**
- **expressing probability as odds**

BACKGROUND INFORMATION:
Probability is a key concept in modern mathematics and in the modern world. Estimating the probability of various events can be of vital importance for the solution of technical, economic, social-political, and other problems. It is fair to say that a recognition of the probabilistic nature of most observed events is one of the foundations of modern culture. The mathematical theory of probability was created relatively recently. Its rudiments first appeared in the 16th-17th centuries, while a strict axiomatic theory was constructed in the 20th century. While the concept of "probability" is familiar to students from their early childhood, formulating a strict definition of it remains rather difficult.

BEFORE SHOWING THE VIDEO:

It is useful to draw the students' attention to the fact that the video will address a very important concept that they often encounter in their daily lives. Describe situations in which it would be useful to learn to estimate the probability of what will happen: What is the probability that it will rain tomorrow? What is the probability that the Mets will win tonight's game? What is the probability that the price of certain stocks will go down by 50%? And so on. The students can be asked to give examples of expressions that they have encountered which contain the word "probability."

CONTENT OF THE VIDEO:

The key concept explained in the video is the probability of an event happening expressed in several different ways. First, the video introduces the concepts of certain and impossible events. Then it defines the probability of an event happening that is neither equal to zero (impossible) nor 100% (certain). The discussion revolves around a basket containing 4 dark-colored and 6 light-colored shells, and seeks to determine the probability of randomly drawing a dark shell. Initially an experimental (statistical) approach is taken. Several experiments with increasingly large numbers of trials all with replacement are conducted, and for each trial, the total number of dark and light shell outcomes is recorded and the probability of a dark shell being drawn out of all possible outcomes, the sample, is calculated. Then the discussion switches to a theoretical calculation of probability. There are 4 dark shells and six light shells, 10 shells in all, the total number of possibilities in the sample space. So theoretically, with replacement, in 4 out of 10 outcomes, a dark shell will be drawn. Consequently, because all of these outcomes are equally likely, the theoretical probability of a dark shell being drawn is $\frac{4}{10} = \frac{2}{5}$. It is emphasized that, with a sufficiently large number of trials, the experimental answer must be close to the theoretical one. In conclusion, the video discusses another way of expressing probability: using odds, i.e. expressing probability as a ratio of a given event happening to that event not happening. For example, in the case examined above, the odds are 4 that a dark shell will be drawn to 6 that it will not be drawn, expressed as the odds being 4:6 (4 to 6) that a dark shell will be drawn. A probability of 4 out of 10 that an event will happen can be expressed as the odds being 4 to 6 that the event will happen.

AFTER SHOWING THE VIDEO:

The students can be given the following problems:

1. Name some certain events, some impossible events.
2. Do the following statements seem right to you? (a) The Mets will win with a probability of 200%. (b) The probability that I will get an A on the test is 1% -- there is no way it can happen!

3. A box has 8 white paper clips and 5 black ones. What is the probability that a randomly selected paper clip will be white?

(a) The students can be asked to determine this probability experimentally. The teacher can break them up into groups, give each group such a box with 8 white paper clips and 5 black ones, and ask each group to conduct 10 trials, with replacement. Then the results obtained by each group individually, and the results obtained by all the groups collectively, can be compared.

(b) This problem can also be solved theoretically -- without experiments.

4. A class has 14 girls and 15 boys. What is the theoretical probability that a randomly selected student will be a boy? What are the odds?

5. The students close their eyes and select one letter from the word PROBABILITY. What is the probability that they will pick (a) the letter P? (b) the letter B? (c) a vowel?

It is useful to stress that the "theoretical approach" which was used in, say, # 3 is based on the fact that all of the paper clips are equally likely to be chosen, therefore all 13 outcomes are equally likely. The teacher can explain that, for example, the probability of a tossed coin coming up heads is $\frac{1}{2}$, since two outcomes are equally likely. But in ancient times crooks used to make special coins for gambling with a displaced center of gravity. In such cases, the two possible outcomes of a coin toss were not equally likely, and therefore, the probability was different from $\frac{1}{2}$.

The students can also be given the following problems:

1. Convert these odds to probabilities.
   (a) 5 : 7 (b) 3 : 8

2. Convert these probabilities to odds.
   (a) $\frac{3}{11}$ (b) $\frac{7}{9}$

EXPLORING AND INVESTIGATING:

The teacher asks the students, if someone randomly throws 2 cubes, each side numbered from 1 to 6., what is the probability that the cubes will total 2? Then the teacher asks if an answer: of $\frac{1}{11}$ could be correct, because all numbers from 2 to 12 are possible outcomes (11 in all), and only one of them is the desired outcome. Students may be asked to explain whether they agree with this answer, and if not, then why not and how they would calculate the correct answer.

Math Series 1, consists of 10 videos:

ALGEBRA: A Piece of Cake Part 1
ALGEBRA: A Piece of Cake Part 2
SLOPES: That's a Bit Steep!
PERCENTAGES: That Make Sense
LINEAR EQUATIONS and Their Graphs: Let's Get It Straight Part 1
LINEAR EQUATIONS and Their Graphs: Let's Get It Straight Part 2
INTEGER OPERATIONS: Into the Negative Zone Part 1 Adding and Subtracting
INTEGER OPERATIONS: Into the Negative Zone Part 2 Multiplying and Dividing
FACTORING IS FANTASTIC Part 1: Common Factors
FACTORING IS FANTASTIC Part 2: Quadratic Trinomials

Math Series 2, consists of 12 videos:

PROBABILITY, Parts 1 & 2
RATIOS
TRIGONOMETRY, Parts 1 & 2
STATISTICS Parts 1 & 2
PROBLEM SOLVING Parts 1 & 2
GEOMETRIC SOLIDS Parts 1, 2, &3